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Seminar über Fragen der Mechanik

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Dynamics of the n-fold pendulum in the framework of Lie group integrators

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The pendulum is a very well-known toy model in multi-body dynamics, whose configuration space is not linear, but a collection of rotations and translations. There are many choices of local coordinates for problems of this kind, whose structure is usually described as a manifold and locally simulated as if it was a linear space. Here we work in the framework of Lie group integrators, which represents a different approach where the model and the numerical integrator are expressed entirely in terms of a Lie group and its action on the phase space. Lie group integrators are widely applied in many study and research areas such as multibody dynamics, shape analysis, data science, image registrations and biophysical simulations.

The mechanical system here is described by the Euler–Lagrange equations on the manifold. We introduce the restriction of the adjoint action of the group $SE(3)$ to TS^2 , the tangent bundle of the two-dimensional sphere. We first give an overview of the case $N = 1$ and then extend the action to the general case $N > 1$ where the phase space becomes $(TS^2)^N$. We then integrate the system with different schemes such as Crouch–Grossman methods, Runge–Kutta–Munthe–Kaas methods and Commutator–free Lie group methods. In the end, we show some numerical experiments where the preservation of certain geometrical properties is in focus.

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